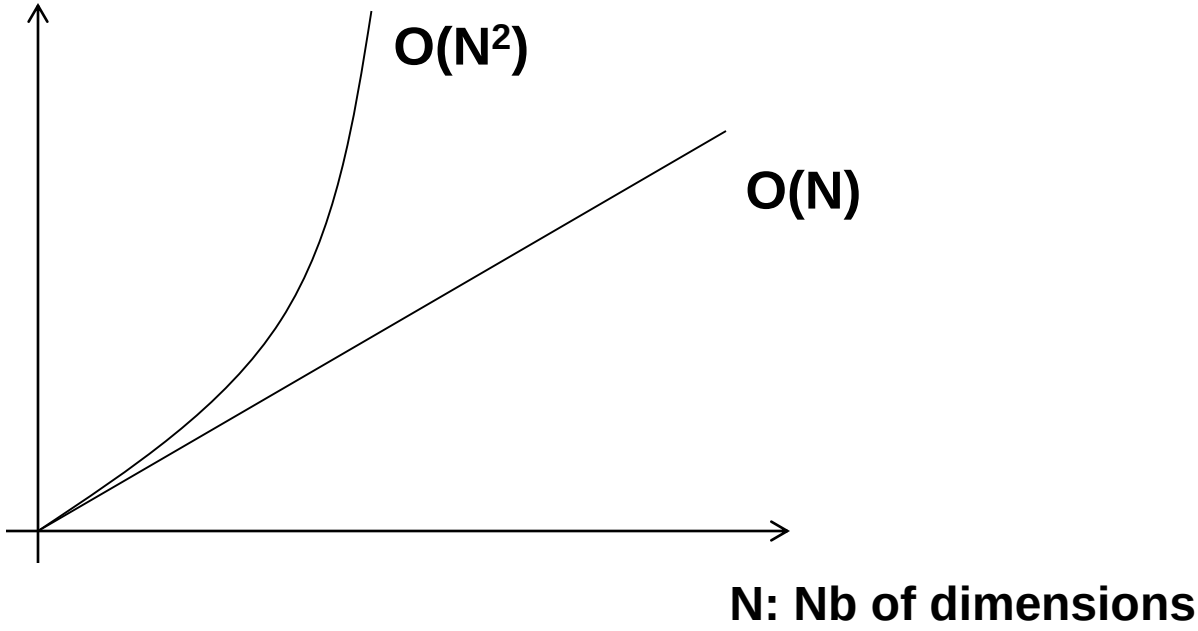
APPLIED MACHINE LEARNING

Principal Component Analysis (PCA)

Part I - Principle

Curse of Dimensionality

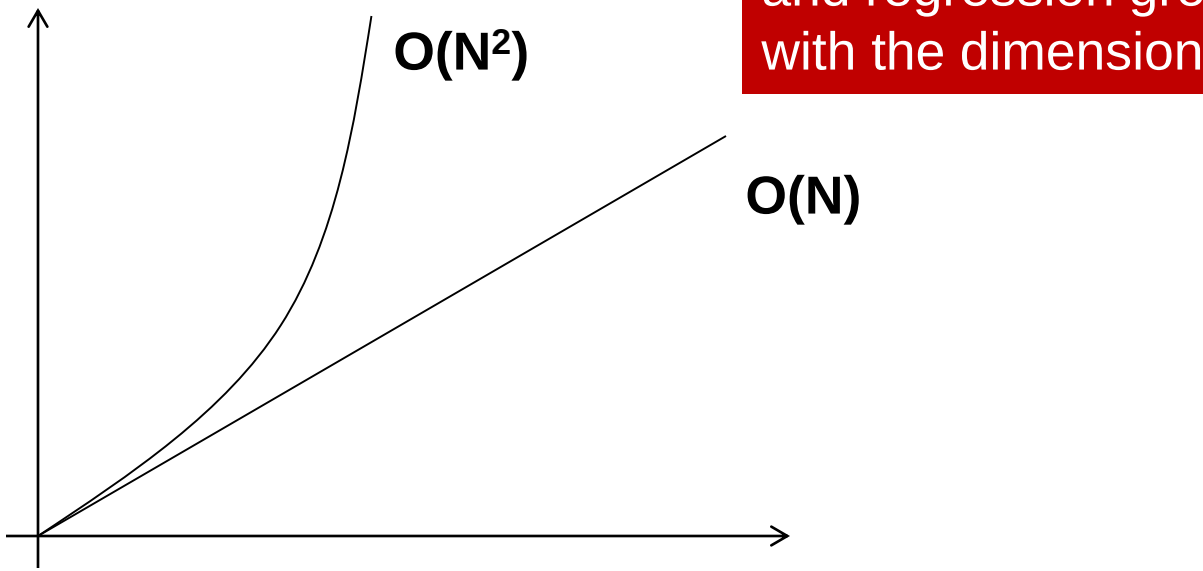
Computational Costs



Linear increase is much preferred over exponential increase

Curse of Dimensionality

Computational Costs



Several methods for classification and regression grow exponentially with the dimension of the data N.

When the increase is exponential/polynomial → reduce dimensionality of data prior to further processing.

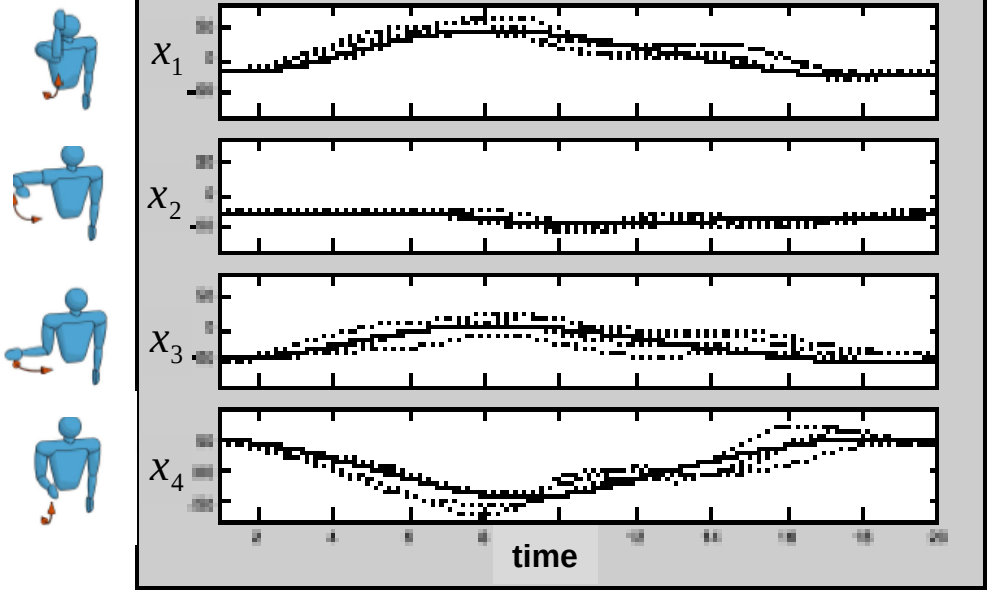
Principal Component Analysis (PCA)

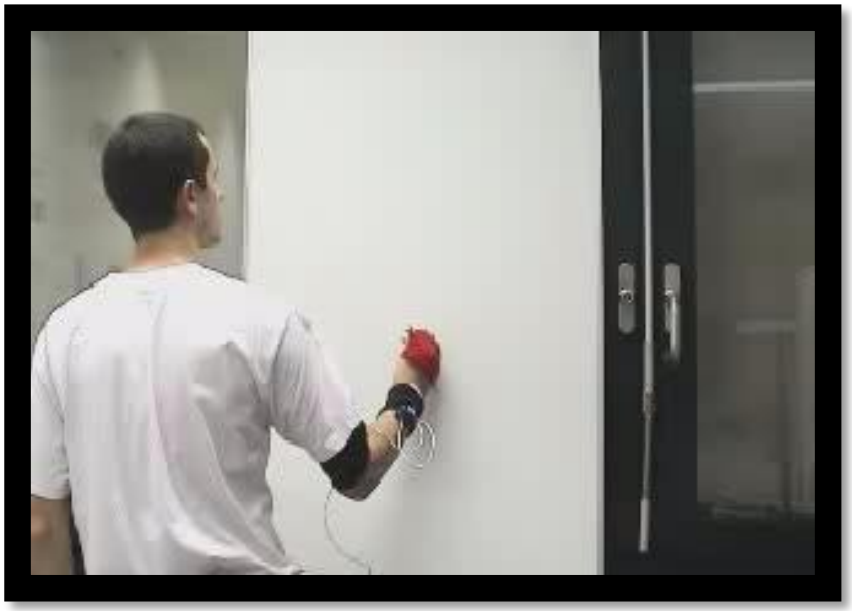
PCA is a method to reduce the dimensionality of dataset.

It does so by *projecting* the dataset onto a lower-dimensional space.

Examples: PCA – dimensionality reduction

Record human motion when writing letters A, B and C



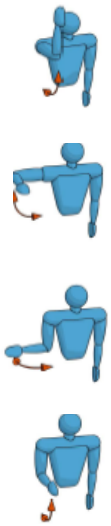


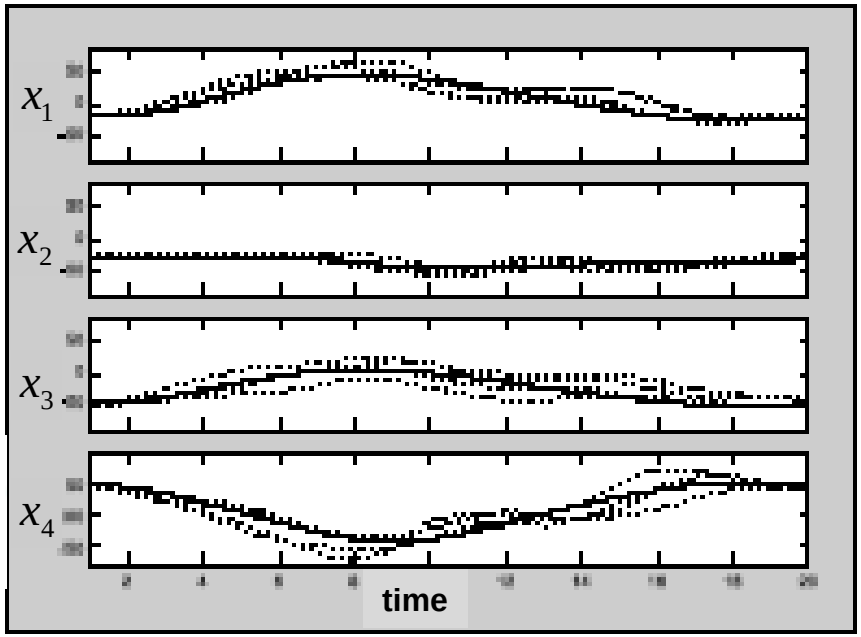
The joint angle trajectories convey redundant information
→ reduce information with PCA

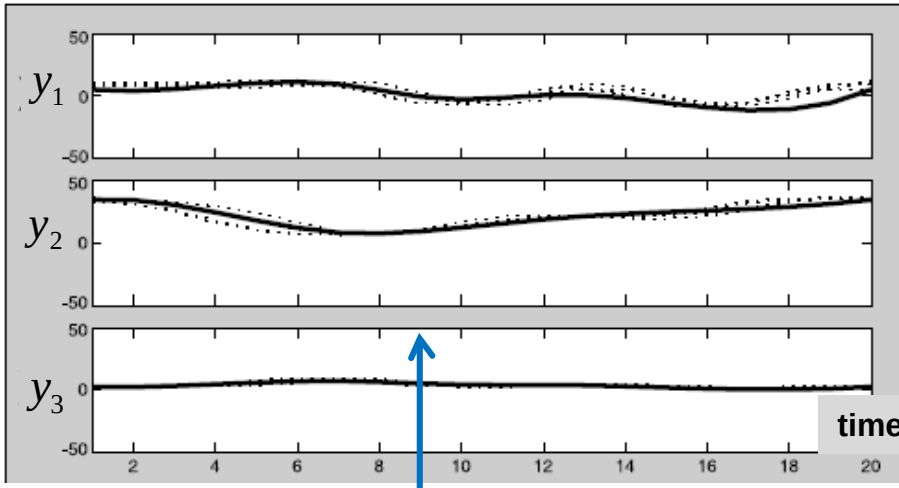
Examples: PCA – dimensionality reduction

$x \in \mathbb{R}^4$ 4-dimensional state space

Project onto 2-dimensional space $y \in \mathbb{R}^3$ through matrix $A \in \mathbb{R}^{3\times4}$

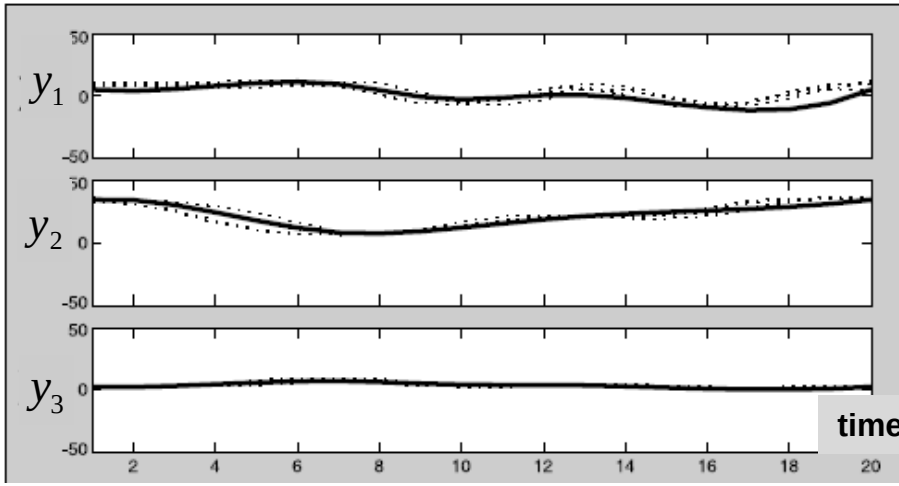






$$y = Ax$$

Examples: PCA – dimensionality reduction



Rotate the projected trajectories in y onto the plane where the robot writes.
Use inverse kinematics to drive the robot's motion.

Examples: PCA – preprocessing for classification

Dataset with samples of two classes (red and green class)

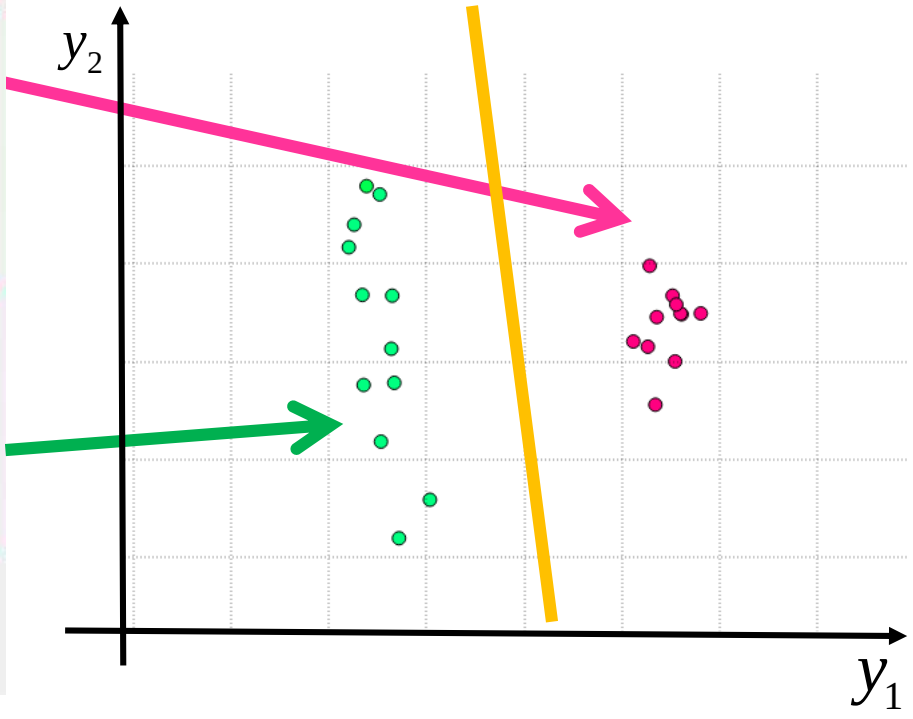
Each image is a high-dimensional vector

$$x \in \mathbb{R}^{320 \cdot 240 \cdot 3 = 230400}$$

Examples: PCA – preprocessing for classification

Project the images onto a lower dimensional space

$y \in \mathbb{R}^2$ through matrix $A \in \mathbb{R}^{2\times 230400}$: $y = Ax$

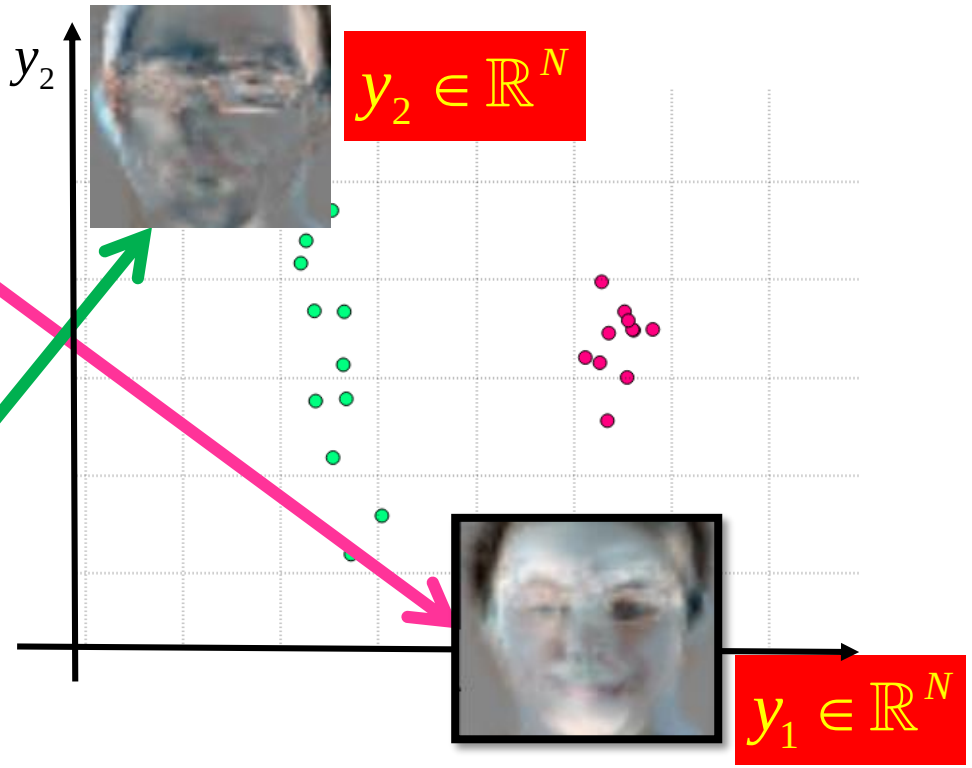


Separating Line

Examples: PCA – preprocessing for classification

Each projection vector y_1, y_2 is N – dimensional.

It is an image!



The projection vectors embed features of the original dataset.

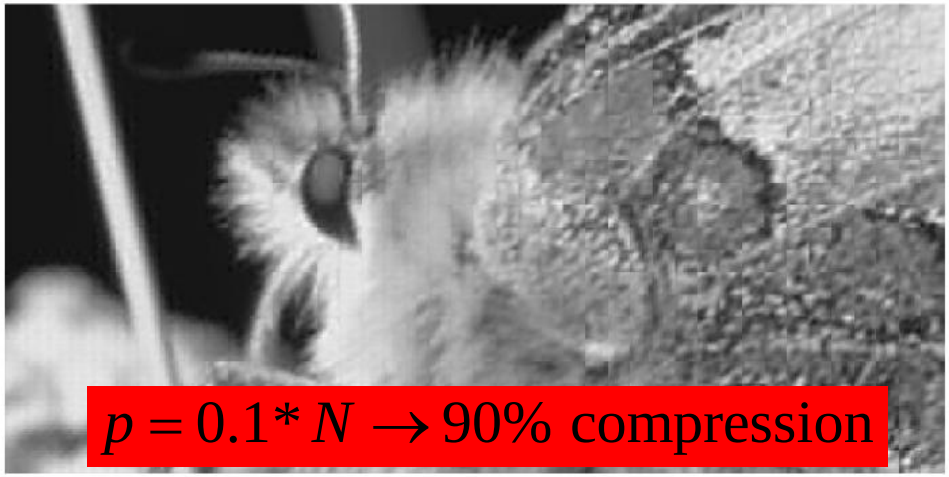
Examples: PCA – preprocessing for data compression

PCA can be used with a single data-point (single image) to reduce the number of dimensions required to represent this data-point.

→ Very useful in processing of high-dimensional images

$x \in \mathbb{R}^N$

Projection $y = Ax, \quad y \in \mathbb{R}^P$



Original image

Image compressed

Principal Component Analysis

PCA has two properties:

1. It reduces the dimensionality of the data.
2. It extracts features in the data.

To achieve 1 & 2, it exploits correlation across datapoints.

PCA can be used as:

1. Compression method for ease of data storage and retrieval.
2. Pre-processing method before classification to a) reduce computational costs, b) extract features to ease classifier's job.